USER BEHAVIOR ON COMPUTING SERVICES
- DERIVATION OF UTILITY FUNCTION -

BY

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Abstract

During the past few years, major emphasis in the field of Computer Economics has been given to technology-cost analysis -- in which cost was analyzed as a function of network topology, transmission technology, and reliability. Recently, the focus of research in this field seems to have changed from the "engineering-economics" point-of-view to the rather "pure economics" point-of-view. This report concerns itself with the purchasing behavior of those buying computing services. Software and hardware are considered commodities.

Chapter 1 is a theoretical section devoted to the derivation of a utility function. This chapter includes an analysis of the behavior of the user of computing devices, mainly -- terminals, small computers and large computers. Determination of utility levels among different users may be considered a field of application.

Chapter 2 is an application of the theory developed in Chapter 1. The network user behavior under certain conditions is analyzed.

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INTRODUCTION

During the past few years, a major effort in the field of Economics of Computers has been given on technology-cost analysis in which cost was analyzed as a function of the network topology, transmission technology, and reliability. Recently, the focus of research in this field seems to have changed from engineering-economics to "rather pure economics point of view." A general study of the effects of pricing on user, Decision Theory under Uncertainty, and the behavior of buyers and sellers of computing services in a market have been studied largely by a group at Stanford. This article concerns itself with the behavior of buyers of computing services in which both software and hardware are considered as commodities.

Chapter 1 is a theoretical section devoted to the derivation of a utility function. This chapter includes an analysis of user behavior on computing devices--mainly, terminal, small and large computers[1]. Determination of utility levels among different users may be considered as one field of application of this chapter.

Chapter 2 is an application of the theory developed in Chapter 1. Analysis of network user behavior is the main topic. This chapter also evaluates the marketability of computer networks in terms of utility.

1 The conventional definition for small computers and large computers will be used here. Small computers are assumed to include the medium sized computers.

CHAPTER 1

1-1. USER BEHAVIOR ON COMPUTER DEVICES

To analyze the user behavior on computing devices, we use some simplifying assumptions that will not distort the crucial aspects of economic reality.

Basic Assumptions

1. The user will spend his limited money (budget) in a way such that he maximizes his utility.
2. There exists three kinds of computing devices: terminal, small and large computers.
3. There exists some levels of utilities of the computing devices such that $U(T) \leq Tm$ for all T, $U(S) \leq Sm$ for all S, $U(L) \leq Lm$ for all L where $U(T)$, $U(S)$ and $U(L)$ are utility functions of terminal, small and large computers, respectively. Tm, Sm, and Lm are the maximum levels of utilities of each device. T, S, and L are the numbers of terminals, small computers and large computers, respectively.
4. Tm, Sm, and Lm have the following relation:

$$Tm < Sm < Lm$$

5. Terminal and small computers are assumed to be substitute good. Small computers and large computers are assumed to be substitute good[1], but terminal and large computers will be assumed to be in an independent relation[2].

---

[1] Substitute good is defined as $\frac{\partial Sq}{\partial Tp} > 0$, where Sq is the quantity of small computers and Tp is the price of the terminal.

[2] Independent relation is defined as $\frac{\partial Lq}{\partial Sp} = 0$, where Lq is the quantity of large computers and Sp is the price of small computers.

1.2 USER'S UTILITY FUNCTION

Definition

From the above assumptions, we can define the user's utility function as follows:

$U_1 = f(T,S)$ where T is the number of terminals and S is the number of small computers.

$U_2 = f(S,L)$ where S and L are the number of small computers and large computers.

The indifference curve[1] of the utility function is defined as $f(T,S) = C$ where C is a constant. An indifference map[2] is generated by allowing C to assume every possible value.

---

[1] An indifference curve is a locus of points--combinations of commodities--each of which yields the same level of total utility, or to which the user is indifferent. The indifference curve have the following characteristics:

a) indifference curves are negatively sloped;
b) an indifference curve passes through each point in the commodity space;
c) indifference curves cannot intersect; and
d) indifference curves are concave from above.

[2] Since U_2 has the same characteristic as U_1, we will only analyze U_1.

1.2.1 Indifference Map

From the definition and characteristics of the indifference curves, the indifference curve and indifference map have the following shapes:

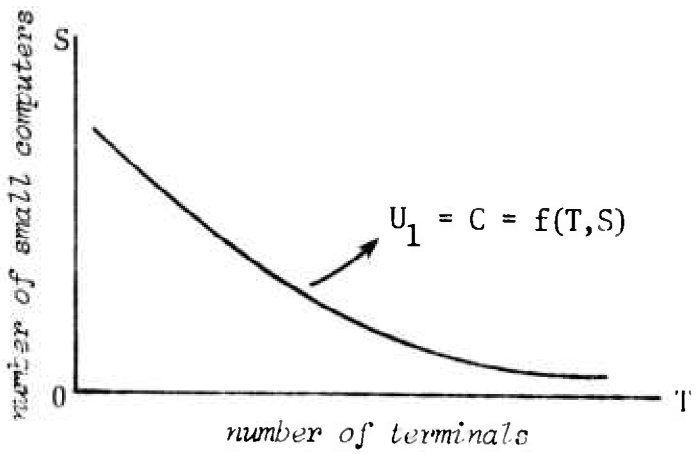

Figure 1

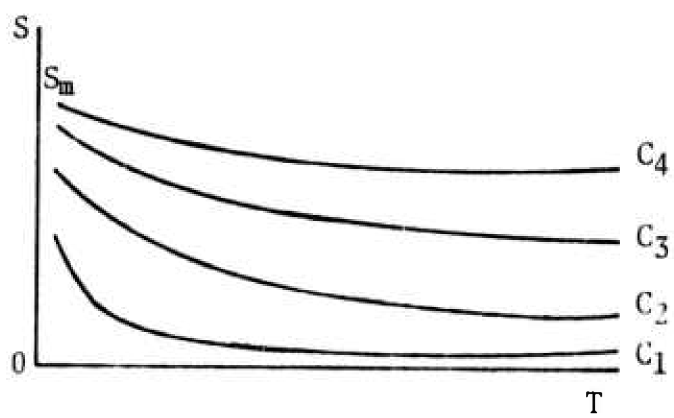

Figure 2

With this indifference map and budget line[1], we can find the optimal point which gives maximum utility.

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[1] Budget line is a line which shows the combination of terminal and small computers that can be bought with a given fixed budget.

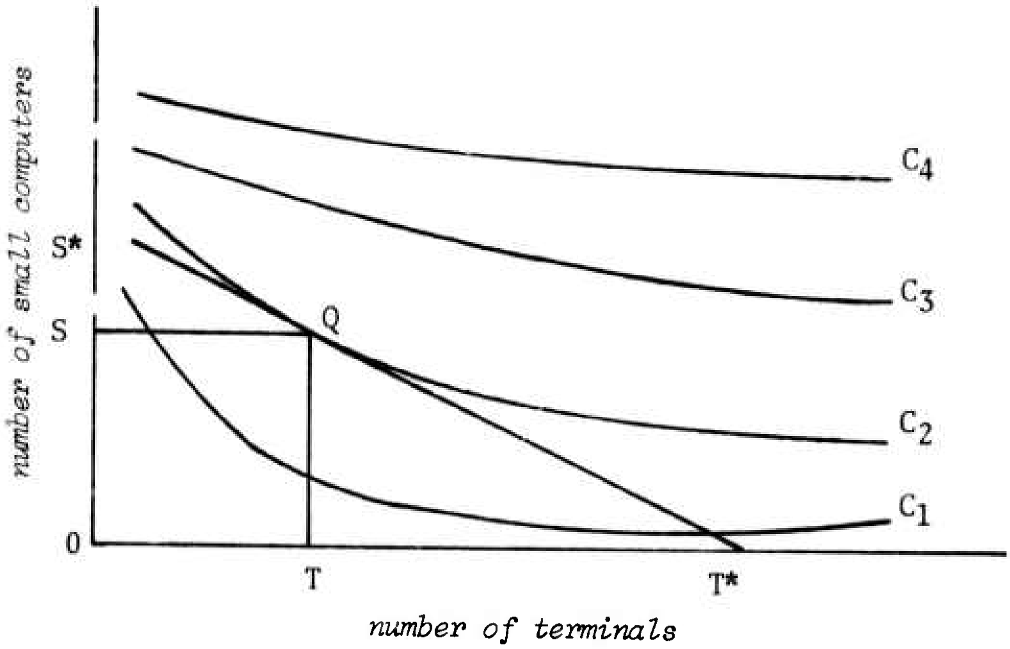


Figure 3

In the above figure, Q is the optimal point at which the consumer will buy OT terminals and OS small computers. Also, at point Q the consumer can get maximum utility, C_2.

1.2.2 Income-Consumption Curve

An increase in budget for computing devices shifts the budget line upward and to the right, and this movement is a parallel shift because nominal prices are assumed to be constant.

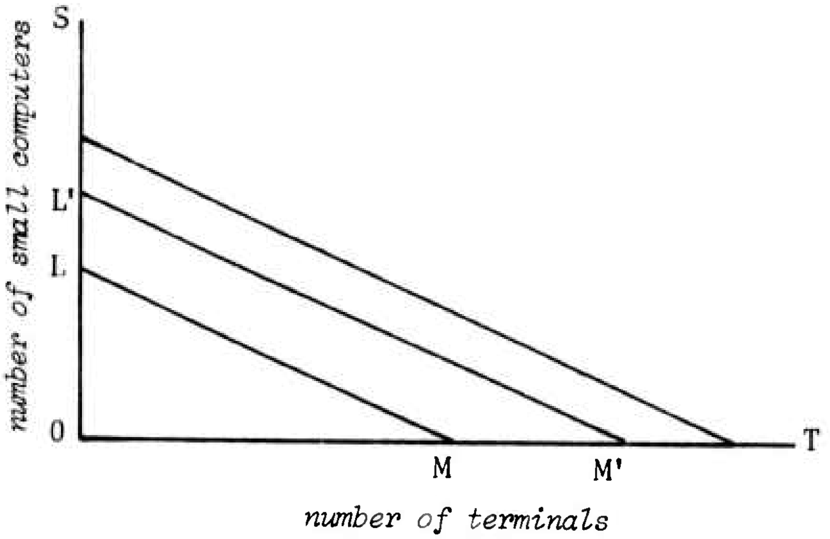


Figure 4

Note: The price ratio is given by the slop of LM. The slope of the original budget line remains constant throughout.

Using the budget line and indifference map, an income-consumption curve for terminal and small computers is derived.

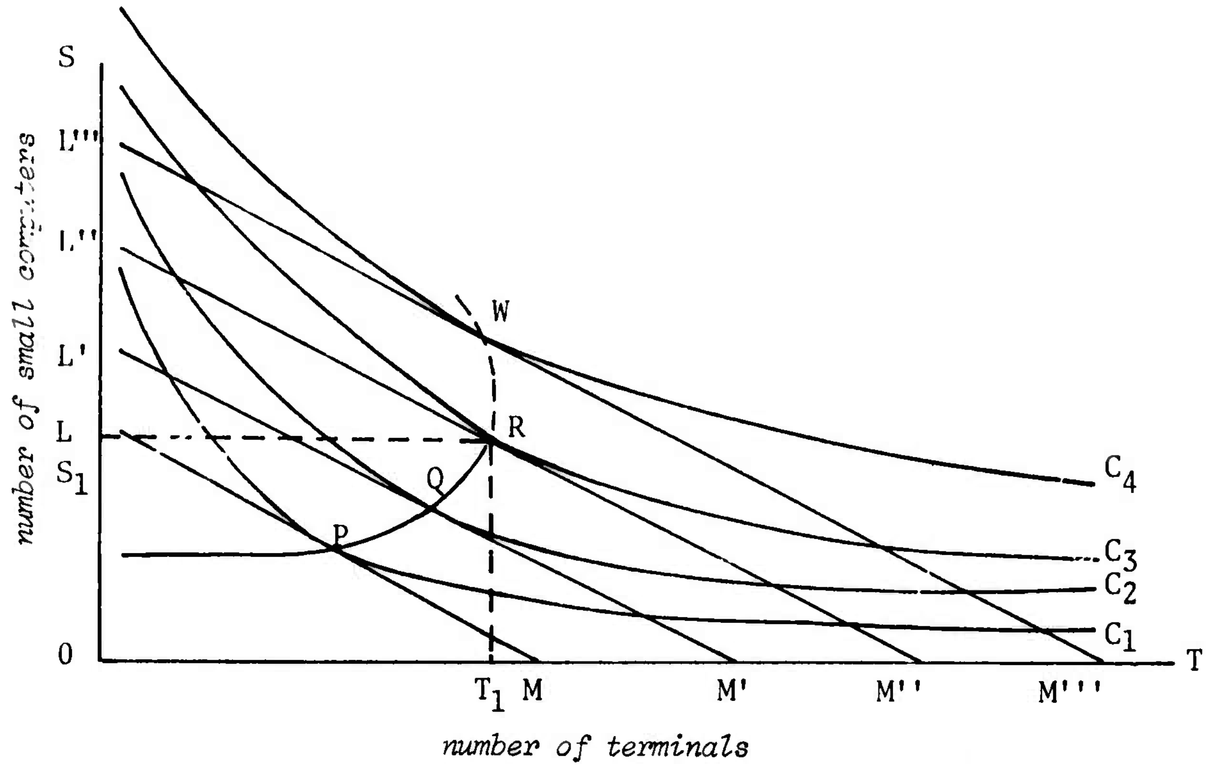


Figure 5

When the budget line rises to the levels represented by L'M', L''M'' ..., the consumer shifts to new equilibriums at Q, R, W..., whose locus is called the income-consumption curve.

Characterisitcs of the income-consumption curve:

1. As the budget for computing increases, the consumer will buy more terminals as well as small computers up to a certain point. The maximum number of terminals demanded is represented as OT1 in Figure 5.
2. When the budget exceeds a certain point, (i.e. R of Figure 5), the user will stop buying terminals by Assumption 3.
3. The utility of the point, (i.e. C3 of Figure 5), implies Tm which is the maximum level of utility of T in Assumption 3.
4. When the budget increases beyond the point, small computers will not compete with terminals but with large computers.
5. By the same inference, we can derive the income-consumption curve for small computers and large computers:

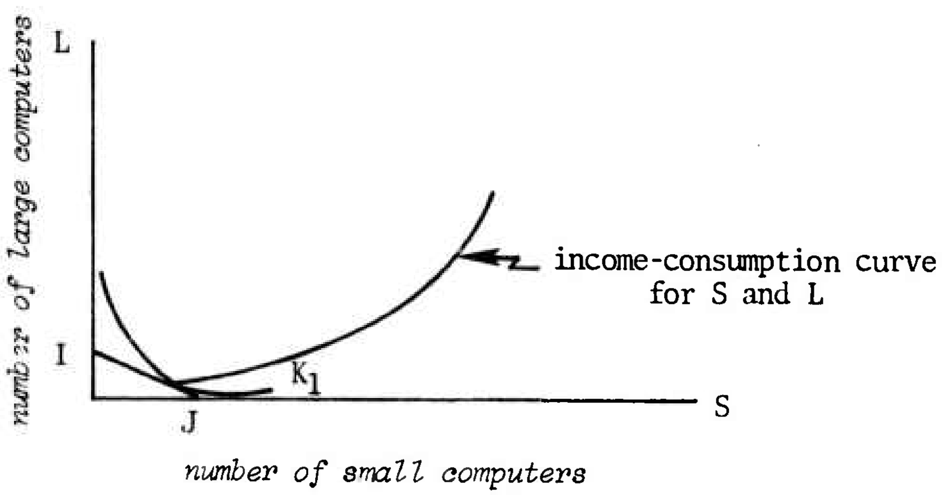
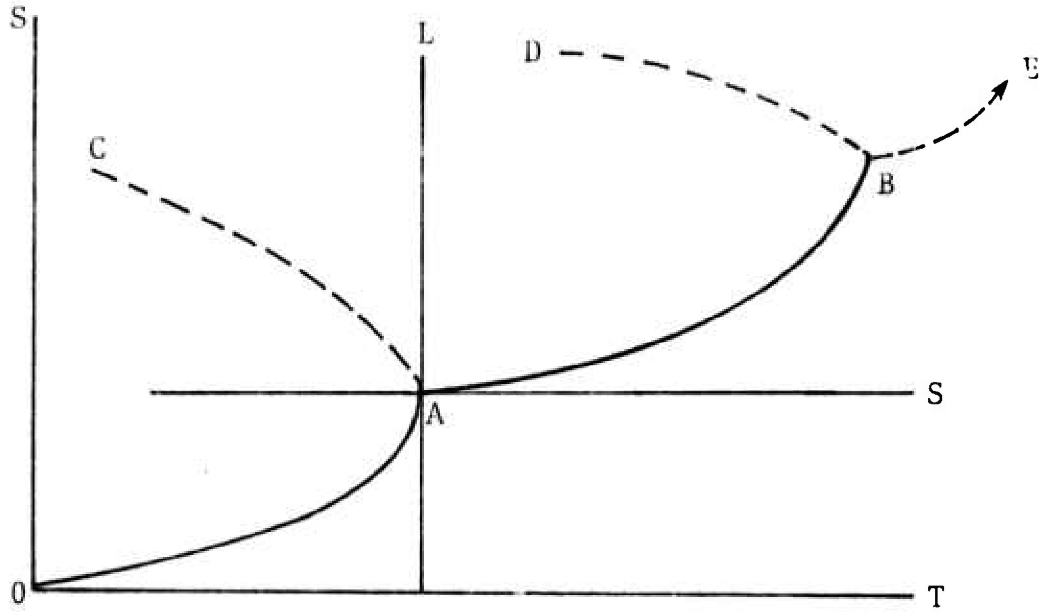


Figure 6

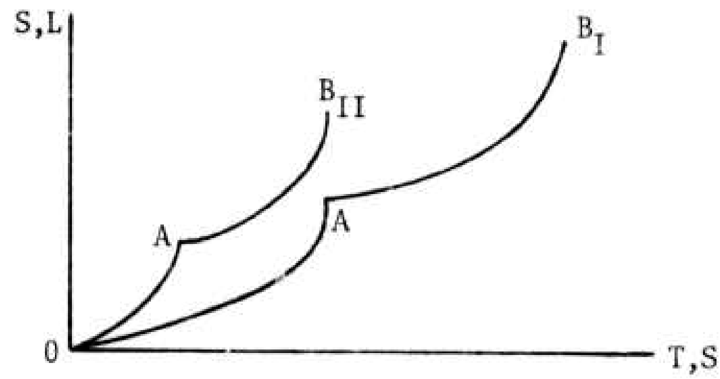
where the budget level IJ is greater than L''M'' of Figure 5 and utility K1 is greater than utility C4 of Figure 5. Therefore, as the budget increases the consumer will behave along the income-consumption curves in the following way:



In this case, if a government does not allow the consumer to buy large computers, which we may find in some developing countries, then at point A the consumer will start to replace terminals with small computers as the budget increases and eventually reach C which has very few terminals.

At point B, if there is no alternative but for large computers, the consumer will take the path to D and eventually replace all small computers with large computers as the budget increases. However, if there is an alternative (say, a large computer network), then the consumer may take path to E.

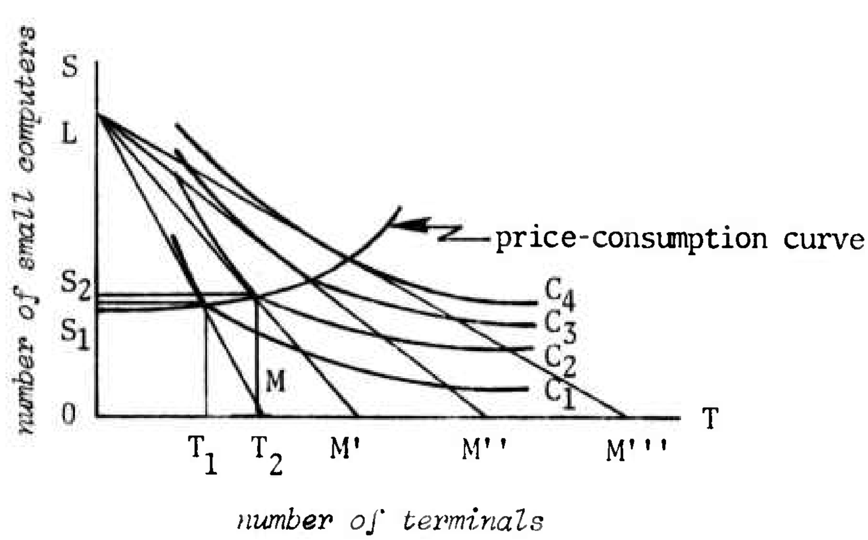
6. The more favorable the price ratio is for S (small computer), the sooner point A is reached. In the same way, the more favorable the price ratio is for large computers, the sooner point B is reached.



Note: The price ratio of II is more favorable than that of I for S to T and L to S.

1.2.3 The Price Consumption Curve

Changes in the price (nominal) of computing devices will also have an impact on consumer behavior. The price-consumption curve represents the locus of equilibrium budgets resulting from variations in the price ratio.



The shift of the budget line from OM to OM' in the above figure implies the decline of the price of terminals. A downward shift of the price of terminals will bring the user a higher level of utility ($C_2 > C_1$) and a greater demand of S and T (from OT1 to OT2, and from OT2 to OT3...).

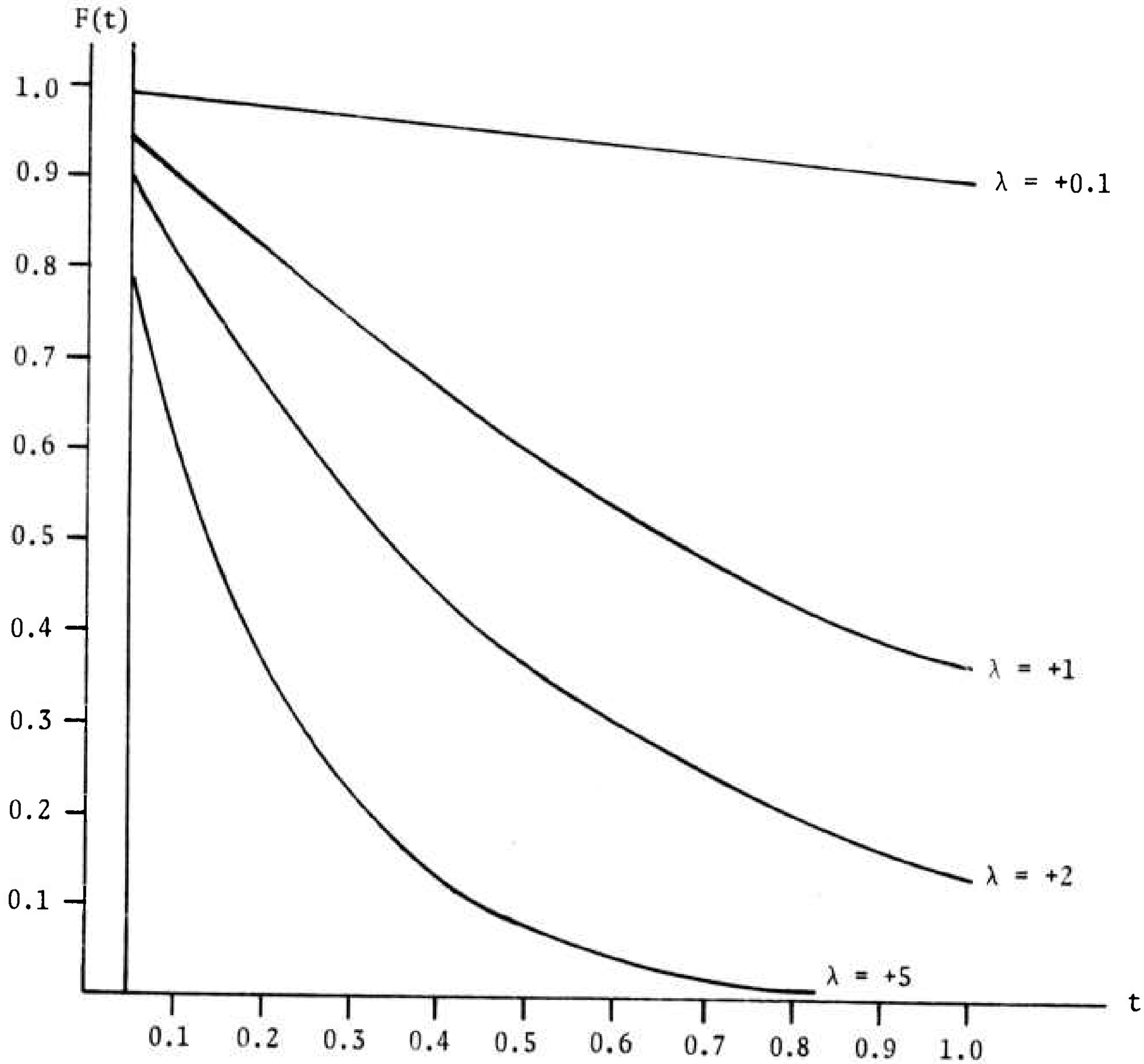
1-3. EXPONENTIAL DISTRIBUTION FUNCTION AS A UTILITY MEASUREMENT

Thus far we have assumed that we know the true shape of the indifference curve which in our case has the same meaning as the utility curve. Suppose we can represent the utility curve as a function of the number of small computers (S) and the number of terminals (T). We could then assign a relative utility level to a user of computing devices.

In an attempt to derive a formula that assigns a utility level to a user, we will consider the characteristics of an exponential distribution function which will then be compared to the properties of the indifference curve discussed in the previous section.

Characteristics of Function $F(t) = e^{-\lambda t}$

1. F(t) has the following map with variations of λ:

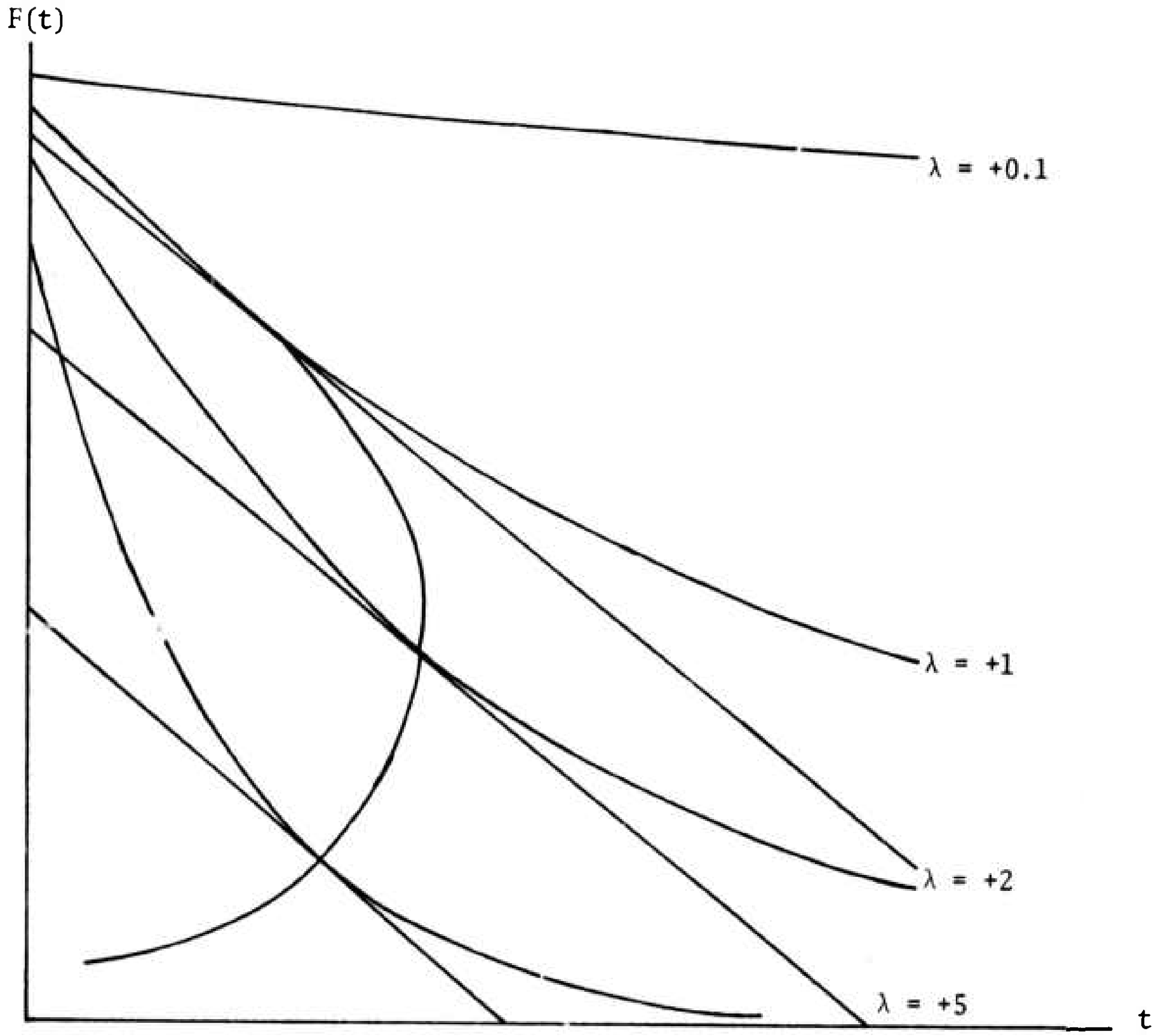


As the value of λ increases (or technically, the value of -λ decreases), the graph of the function moves closer tc the origin.

2. At $t = 0$, $F(t) = 1$ for all λ.

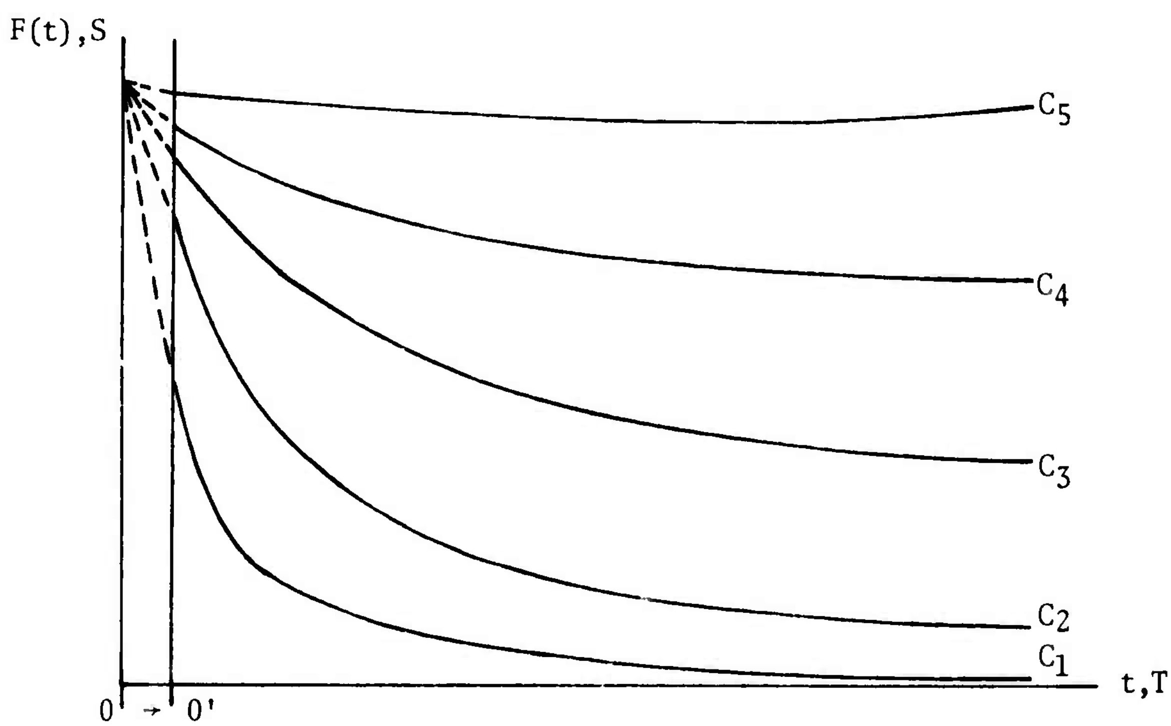
3. At $\lambda = 0$, $F(t)$ has the maximum value, i.e., 1 for all t.

4. Given a fixed value k, the locus of the points $F(t)$ for all λ such that $\frac{\partial(F(t))}{\partial t} = k$ (for all λ, t = any number except 0) is represented in the following graphic form.



We see from the above characteristics that the function F(t) has many similar characteristics to that of the utility function whose $U_1 = f(T,S)$ (or $U_2 = f(S,L)$). Suppose we redefine and reinterpret the function F(t), that is, t as T and F(t) as S, then the consumer's utility function can be represented as $\mu = 1/(-\lambda)$ where $\mu = 1/-\lambda = T/\ell nS$ [1] where T is the number of terminals and S is the number of small computers. The utility function does not have any meaning at T = 0 since, at T = 0, $U_1(T,S)$ is zero. That is, $U_1 = 1/-\lambda = T/\ell nS$. At T = 0, $U_1 = 0$. The characteristics of the indifference curve also does not allow a utility function at T = 0, since at that point all $-\lambda$'s have the same value violating Characteristic 3 of the indifference curve.

Hence, we have to shift the origin slightly to the right[2].



1 $F(t) = e^{-\lambda t} \rightarrow \ell nF(t) = -\lambda t \rightarrow -\lambda = \ell nF(t)/t$. Since we assumed F(t) as S and t as T, $-\lambda = \ell nS/T$. Hence, $\mu_1 = 1/-\lambda = f(T,S) = T/\ell nS$.

2 Since we assumed an ordinal utility function, it is enough to shift the origin slightly to the right. The distance of shift is not important in our analysis.

The results of a comparison between the properties of the indifference curve and the exponential distribution function show us that the exponential distribution function has most of the desirable properties of the indifference curve which represents user behavior on computing devices.

From the above, we can derive a utility function U_t such that $U_t = a\frac{T}{\ell n S}$, where U_t is the utility level of the combination of T and S. T is the number of terminals, S is the number of small computers and a is a coefficient. The value of a is not important since we are dealing with ordinal utility. But for convenience, a will be assumed as 1.

In the same way, utility function U_s can be represented as $U_s = b\frac{S}{\ell n L}$, where L is the number of large computers. (Here, b will be assumed as 1 also.)

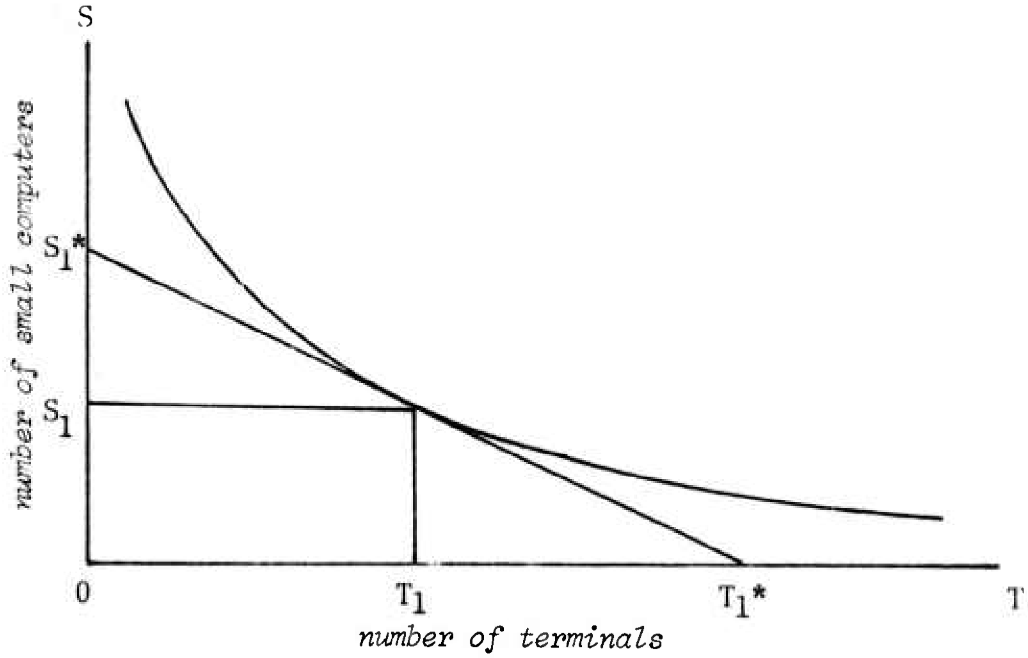
Using this formula, we can assign utility levels to users who might have different values of S, T, and L.

1.3.1 Utility Level as a Function of Price Ratio

Before obtaining a formula for the utility level as a function of price ratio, let us define some of the basic concepts.

$$\text{Price ratio} = \frac{1 \text{ (unit number of small computer)}}{\text{number of terminals equivalent to unit number of small computers}}$$

$$\text{Quantity ratio} = \frac{\text{number of terminals at a specific point}}{\text{number of small computers at a specific point}}$$



In the above figure, OT1/OS1 represents the quantity ratio and $\frac{1}{\frac{OT1^*}{OS1^*}}$ represents the price ratio.

Assuming the utility level as μ we can derive a formula to measure utility in the following way:

$$\mu = T/\ell nS \quad \text{where } T > 0$$

$$\mu \ell nS = T$$

$$S = e^{T/\mu} \tag{1}$$

$\frac{\partial S}{\partial T} = \frac{1}{\mu} e^{T/\mu}$ where $\frac{\partial S}{\partial T}$ is tangent at the specific point, and has P_T as its value. (P_T is the number of terminals that can be bought by one small computer at market price.)

$$\frac{\partial S}{\partial T} = \frac{1}{\mu} e^{T/\mu} = P_T$$

$$e^{T/\mu} = \mu P_T$$

$$T/\mu = \ell n\mu + \ell nP_T$$

$$T = \mu(\ell n\mu + \ell nP_T) \tag{2}$$

$\frac{T}{S}$ is the quantity ratio at the specific point. Therefore,

$$\frac{S}{T} = \frac{1}{Q_T} \qquad (3)$$

where Q_T is the number of terminals per small computer that are actually bought ($Q_T = T/S$).

Substitute (1) and (2) into (3):

$$\frac{e^{T/\mu}}{\mu(\ell n\mu + \ell nP_T)} = \frac{1}{Q_T}$$

$$\frac{e^{\mu(\ell n\mu + \ell nP_T)/\mu}}{\mu(\ell n\mu + \ell nP_T)} = \frac{1}{QT}$$

Finally we get[1]

$$U = e^{e^{\ell nQ_T + \ell n(\frac{P_T}{\ell nP_T})}} = e^{Q_T P_T/\ell nP_T}$$

U, which is equivalent to U_t in the previous equation, i.e., $U_t = T/\ell nS$, is the utility level which is decided by Q_T and P_T where Q_T represents the number of terminals per small computer that are actually bought and P_T is the

---

1 $$\frac{e^{\mu(\ell n\mu + \ell nP_T)/\mu}}{\mu(\ell n\mu + \ell nP_T)} = \frac{1}{Q_T}$$

$$Q_T e^{\ell n\mu + \ell nP_T} = \mu(\ell n\mu + \ell nP_T)$$

$$\ell nQ_T + \ell n\mu + \ell nP_T = \ell n\mu + \ell n(\ell n\mu + \ell nP_T)$$

$$\ell nQ_T + \ell nP_T = \ell n\ell n\mu + \ell n\ell nP_T$$

$$\ell n\ell n\mu = \ell nQ_T + \ell nP_T - \ell n\ell nP_T$$

$$\mu = e^{e^{\ell nQ_T + \ell nP_T - \ell n\ell nP_T}} = e^{Q_T P_T/\ell nP_T}$$

number of terminals that can be bought by one small computer at market price.

Characteristics of Utility Level

As we can see, the utility level μ will increase when Q_T increases or P_T increases. The indicator of price ratio, P_T, is given to the user from the market. Therefore, the main variable determining utility level is Q_T, the indicator of individual quantity ratio..

That is, if we know Q_T or the desirable ratio of Q_T of a user, we can measure his utility level. In reality, Q_T is defined by price ratio. Therefore, we can represent Q_T as a function of P_T. (Main decision variable to a user in buying terminals is the price of a terminal in the market.) Then the utility function is defined as follows:

$$U = e^{f(P_T)} \cdot \frac{P_T}{\ell n P_T} \quad (1)$$

In a competitive market, an individual Q_T cannot affect P_T. However, in the long run, an individual Q_T tends to follow P_T. Suppose $Q_T = P_T$, then we have:

$$U = e^{P_T{}^2/\ell n P_T}$$

This is what I will call a <u>social utility function</u> (or long-run utility function). This determines the market boundary condition for a specific commodity.

As a special case, suppose $P_T = 1$.

$$U = e^{P_T{}^2/\ell n P_T}$$

$$= e^{1/0} \quad \text{which is indeterminate.}$$

[1] Since we assumed a unit number of small computers in deriving this utility level, we may use U as the utility level of terminals.

Suppose also that $P_T < 1$, then

$$U_T = e^{P_T{}^2/\ell n P_T}$$

where $\ell n P_T < 0$, which cannot be determined and therefore U_T cannot be determined. Therefore, condition $P_T > 1$ is defined as a market boundary condition for T which is a terminal in our case.

Another characteristic of this utility function is that we can have the same utility level even if we have different absolute values of S and T, since P_T is a relative value and not an absolute value. In other words, the price ratio is defined as 1 (unit of small computer)/(number of terminals equivalent to 1 small computer in terms of market price) or $\frac{1}{P_T}$.

Thus, as long as both the price of terminal and the price of small computer change at the same rate, the utility level before and after the price change will be the same.[1]

$$\frac{1}{P_T} = \frac{k}{kP_T} = \frac{1}{P_T}$$

Testing of the Utility Function

Previously, we defined the utility function as $U = \frac{T}{\ell n S}$. From this, we can get $T = U \cdot \ell n S$. It means that T is proportional to $\ell n S$. Therefore, the utility level U can be interpreted as a coefficient of the above equation.

Suppose we have a set of data on T and S from real observations. Then, we can set up a regression model from the data. Let the regression model be:

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[1] However, in the short-run where $U_T = e^{Q_T P_T/\ell n P_T}$, this is not true since Q_T implicitly assumes the budget constraint. In this case, different absolute values of S and T will have different utility levels even if we have the same price ratio.

$$T_i = \alpha + \beta \cdot \ell n S_i + \varepsilon_i$$

where ε_i is the disturbance term. Under the assumptions of the classical regression model[2], we could estimate the value of β and construct the estimated regression equation. Let the following equation be the estimated regression equation:

$$\hat{T}_i = \hat{\alpha} + \hat{\beta} \cdot \ell n S_i + \varepsilon_i$$

where $\hat{T}$, $\hat{\alpha}$ and $\hat{\beta}$ are the estimated value of T, α and β respectively. Here, the true value of $\hat{\beta}$ is the utility level μ, since

$$T = \mu \cdot \ell n S$$

$$\text{where } \mu = e^{Q_T P_T / \ell n P_T}.$$

Using the above relations, we can set up a test statistic for testing whether the utility function represents the true value of the user utility level or not.

We may set the test hypothesis in the following way:

$$H_0: \quad \beta = \mu = e^{Q_T P_T / \ell n P_T}$$

$$H_A: \quad H_0 \text{ is not true}$$

Using t-statistic we have:

$$\frac{\hat{\beta} - \beta}{S_{\hat{\beta}}} = \frac{\hat{\beta} - e^{Q_T P_T / \ell n P_T}}{S_{\hat{\beta}}} \sim t_{n-2} \tag{4}$$

where n is the number of observations, and $S_{\hat{\beta}}$ is a standard deviation of $\hat{\beta}$.

[2] The assumptions of the classical regression model are:

1. $\varepsilon_i \sim N(0,\sigma^2)$
2. $E(\varepsilon_i \varepsilon_j) = 0 \ (i \neq j)$
3. S_i is a nonstochastic variable.

The other testing hypothesis[3] is

$$H_0: \ \alpha = 0$$
$$H_A: \ \alpha \neq 0$$

Again using t-statistic we have

$$\frac{\hat{\alpha} - \alpha}{S_{\hat{\alpha}}} = \frac{\hat{\alpha}}{S_{\hat{\alpha}}} \sim t_{n-2} \tag{5}$$

where $S_{\hat{\alpha}}$ is a standard deviation of $\hat{\alpha}$.
When we accept the null hypothesis of test statistic (4), i.e., $H_0: \ \beta = \mu$, and reject the null hypothesis of test statistic (5), i.e., $H_0: \ \alpha = 0$ under the appropriate testing rule set by each researcher. We may use the utility function as our "true" utility function by which we can assign utility levels to different combinations of T and S.

One problem to be solved here is how to assign the value of Q_T and P_T in computing the utility level, since the value of μ will vary as P_T varies. (Therefore, Q_T also varies.) One simple solution to the problem is to take the expected value of μ for all observations. That is, we may replace μ as $E(\hat{\mu})$, where $E(\hat{\mu}) = E(e^{Q_T P_T / \ell n P_T})$. Then our test statistic of (4) will be:

$$\frac{\hat{\beta} - \beta}{S_{\hat{\beta}}} = \frac{\hat{\beta} - E(e^{Q_T P_T / \ell n P_T})}{S_{\hat{\beta}}} \sim t_{n-2}$$

When our theory is accepted from the two tests, we could derive the demand function of T or S using the utility function and budget constraint. Since this topic is beyond the scope of this report, this report will not include the demand function.

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3 The basis of this test comes from one of the characteristics of the indifference curve, i.e., any two indifference curves cannot intersect each other. From this, we've got $T \neq 0$ condition, which implies $\alpha \neq 0$.

CHAPTER II

2.1 ANALYSIS OF NETWORK USER BEHAVIOR

This chapter is essentially an application of the theory developed in the previous chapter. The network user's behavior in terms of network utility is the main concern of this chapter.

In the first section, we discuss one commodity model in which we assume that only one software is available in a network. Later, we relax our assumption and see how a network user behaves.

Basic Assumptions

The following are basic assumptions:

1. There is a computer network with a user who has his own computer whose size is the same as all other nodes.

2. There is only one commodity, i.e., one software in the network, which is supplied by one supplier of the network.

3. Installing the software requires cost.

4. User has two strategies: One is to install the software in his computer and use it, denoted as R1, and the other is to use the network, denoted as R2. Whether a user uses the network or not determines the boundary condition for a network market.

5. We assume that there is no congestion from the network itself.

We rewrite our utility function: $U = e^k$ where

$$k = f(P_T) \cdot P_T/\ln P_T$$

or

$$k = Q_T \cdot P_T/\ln P_T$$

Let P_T represent the number of times in which a user can use the network with the same amount of money as in the case of using his own computer.

Utility function, U, can be defined as a function of N representing the number of usages of the software.

Notation:

Let us denote the cost of R1 as Cr1 and the cost of R2 as Cr2.

Then C_{R1}, total cost of using his own computer is the sum of the installing cost and running cost. Here installing cost may be defined as a fixed cost, and running cost as a variable cost. Let the installing cost be C_i and the running cost be $C\mu$. C_{R1} can be defined in the following equation:

$$C_{R1} = C_i + C\mu$$

The average cost can be derived by dividing both sides by N which is the number of usages of the software.

$$A_{R1} = C_{R1}/N = C_i/N + C\mu/N$$

Since $C\mu$ is a function of N represented as $C\mu = U \cdot N$, $C\mu/N$ becomes constant as N varies. Therefore, in this case we can say that the average cost is a function of C_i and N.

Hence we can get the following results:

1. As N increases, average cost will decrease to a certain point. Up to that point, economies of scale dominates the average cost but beyond that point diseconomies of scale dominates.

2. The higher the installing cost, the higher the average cost.

C_{R2}, the total cost of using the software through network, is a function of

running cost, congestion cost, and network cost. Congestion cost has the same meaning as delay cost. Network cost includes the costs required in using the hardware of a network (therefore excluding delay cost). Network cost can be considered either as a fixed cost (as in The ALOHA System) or as a variable cost. Denoting network congestion cost as C_{C2} and network cost as C_N, we can rewrite C_{R2} in the following way:

$$C_{R2} = C\mu_2 + C_{C2} + C_N$$

where $C\mu_2$ is the cost of the software service to the user, and can be represented as $C\mu_2 = U_2 \cdot N$, where U_2 is the price of the software service from the supplier.

The average cost can again be derived by dividing both sides by N:

$$A_{R2} = C_{R2}/N = C\mu_2/N + C_{C2}/N + C_N/N$$

Here $C\mu_2$ is constant but $\frac{C_N}{N}$ will vary as N varies, and $\frac{C_{C2}}{N}$ will also vary as N increases. Therefore, in this case, the average cost is a function of C_{C2}, C_N, and N.

As N increases we have the following tendencies in each variable:

C_N/N will decrease if C_N is a fixed cost.

C_N/N will be constant if C_N is a variable cost. (Packet-switching networks is a typical example of this.)

C_{C2} will increase.

Therefore, we can say that the average cost will increase as N increases sufficiently.

2.2 RELATIONS TO UTILITY FUNCTION

Now let us see how these factors affect the utility function.

As we have already discussed, the utility function is a function of price ratio between two alternatives. That is,

$$\text{price ratio} = \frac{C_{R2}/N}{C_{R1}/N} = \frac{C_{R2}}{C_{R1}} = \frac{1}{C_{R1}/C_{R2}}$$

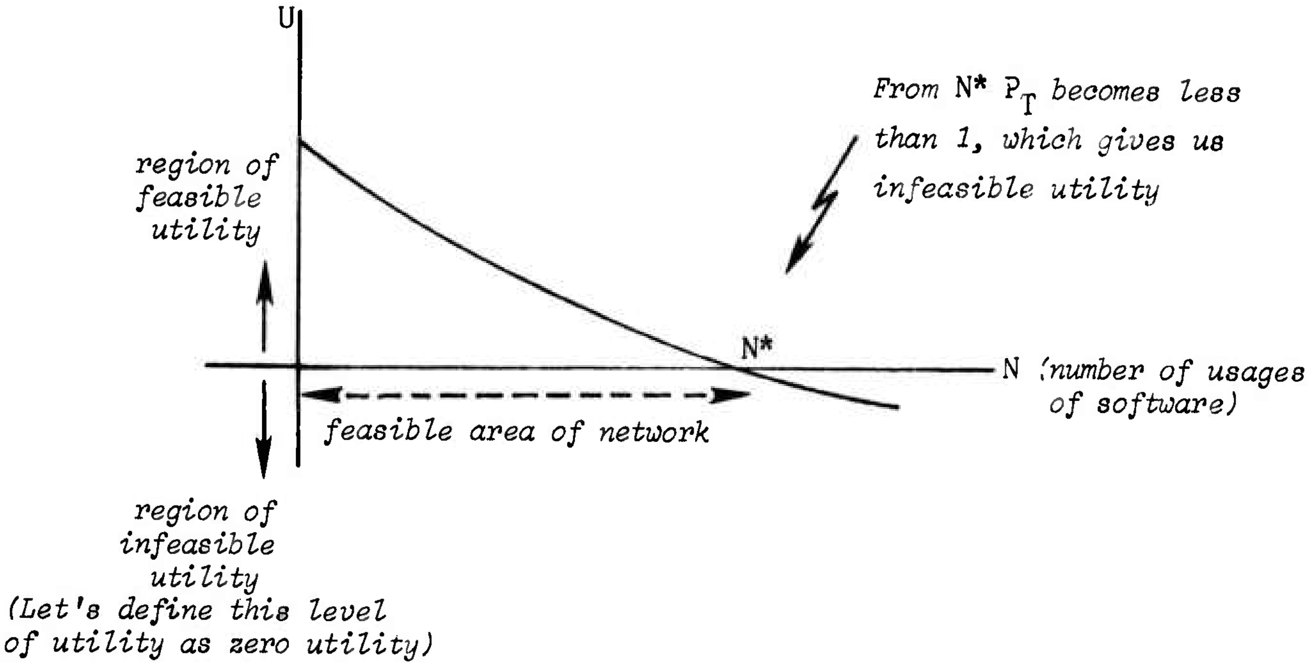
$$= \frac{1}{(C_i+C\mu_1)/(C\mu_2+C_N+C_{C2})} = \frac{1}{P_T}$$

where
$$P_T = \frac{C_i+C\mu_1}{C\mu_2+C_N+C_{C2}}$$

As N increases, $\frac{C_i}{N}$ decreases and $\frac{C\mu_1}{N}$ and $\frac{C\mu_2}{N}$ are constant. And $\frac{C_N}{N}$ is either constant or decreases depending upon whether C_N is a fixed cost or a variable cost.

In the case where C_N is a variable cost, P_T will decrease as N increases since C_N is constant. Therefore the utility of the network will gradually be reduced. Finally, the user will reach a point at which the utility of the network cannot be determinate.

On the other hand, P_T will have a value greater than 1 only when C_i is greater than the sum of C_N and C_{C2}. Suppose we do not have any congestion in using the network. (Assume the network has excellent performance or is within the permissible tolerance level range in the user's point of view.) Then the utility of the network is determined by the difference between the installing cost and the network cost.



If installing cost is greater than network cost, i.e., $C_i > C_N$, the utility of the network is greater than or equal to zero and therefore the user will use the network.

$$P_T > 1 \text{ if } C_i > C_N$$

Thus, $U > 0$. This is true as N gets very large. Thus, if we do not consider congestion cost in the network, and if we have the above condition, then the utility of the network for the user will not be negative. Therefore, he will use the network continuously even though utility decreases gradually.

So far we have obtained very interesting results. That is, if the network cost is constant as N varies, there exists a boundary point, N*, at which the utility becomes negative. Also, the higher the installing cost, the larger the value of N*.

2.3 CONGESTION COST

Suppose we have congestion from computers as the number of usages increases.

$$C_{R1} = C_i + C\mu_1 + C_{C1},\ C_{R2} = C\mu_2 + C_{C2} + C_N.$$

Average cost:

$$A_{R1} = C_{R1}/N = C_i/N + C\mu_1/N + C_{C1}/N$$

$$A_{R2} = C_{R2}/N = C\mu_2/N + C_{C2}/N + C_N/N$$

By definition,

$$\text{price ratio} = \frac{A_{R2}}{A_{R1}} = \frac{C_{R2}/N}{C_{R1}/N} = \frac{C_{R2}}{C_{R1}}$$

$$= \frac{1}{C_{R1}/C_{R2}} = \frac{1}{P_T}$$

Therefore:

$$P_T = \frac{C_{R1}}{C_{R2}} = \frac{C_i + C\mu_1 + C_{C1}}{C\mu 2 + C_{C2} + C_N}$$

Therefore

$$P_T > 1 \text{ if } (C_i - C_N) + (C\mu_1 - C\mu_2) + (C_{C1} - C_{C2}) > 0$$

Again, as N increases:

C_i will decrease,

C_N will be constant (in the case of variable cost)

or decrease (in the case of fixed cost),

$C\mu_1$, $C\mu_2$ are constant (we assumed short-run), and

C_{C1}, C_{C2} will increase.

Since there is only one software in the network, other users may use the software. Consequently, for the same N the degree of congestion in the case of network usage will be greater than in the case of using his own computer. Congestion from the network itself will also accelerate this phenomenon. It gives us the following:

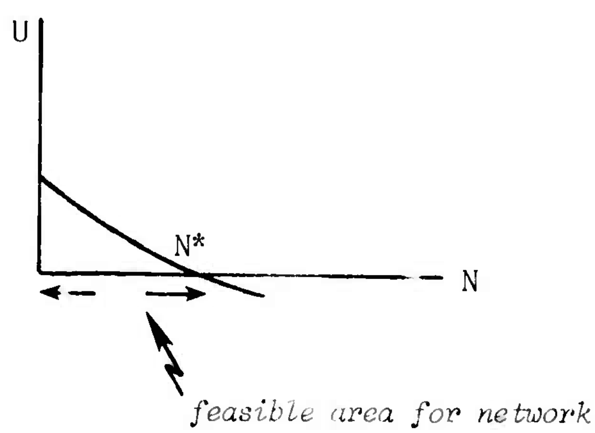
As N increases:

$C_i - C_N$ will eventually be negative or approach zero,

$C\mu_1 - C\mu_2 = 0$ (because we assumed the same size of computer), and

$C_{C1} - C_{C2}$ will be negative.

Thus we can draw the following diagram:



In the case where C_N is a fixed cost, and the difference between C_i (installing cost) and C_N (network cost) is large, there exists a feasible area for the network. But as we have seen in the discussion of the relation between C_i and C_N, the difference will approach zero as N increases and the user will decide not to use the network reaching N* since after N* the utility of the network becomes less than zero.

But if C_N is a variable cost (and since installing cost, C_i, is usually greater than C_N), the feasibility of the network will be determined by the difference between (C_i-C_N) and $(C_{C1}-C_{C2})$.

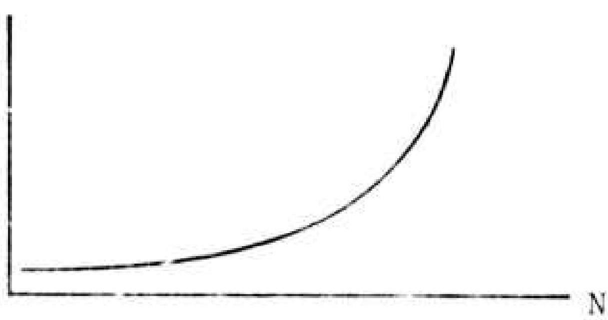
$$U \text{ (utility of network)} > 0 \text{ if } C_i - C_N > C_{C1} - C_{C2}.$$

The higher the installing cost, the larger the feasibility of the network market. Or the lower the variable network cost, the more he will use the network.

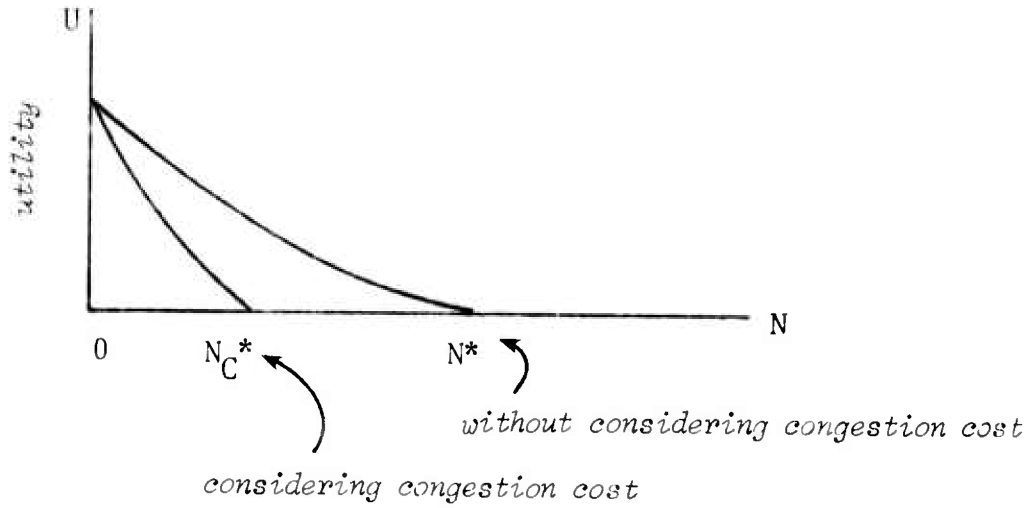
Since in ordinal condition, in which we assume users behave randomly with a mean number of usages of the network, congestion cost rapidly approaches infinity after a certain point,[1] the user will soon reach the point where he prefers his own computer to the network.

The number of nodes is crucial in this case. For smaller number of users the difference between C_{C1} and C_{C2} is negligible. In this case, the user will follow the same behavior as the previous case where we do not consider congestion cost.

[1] Congestion cost of network



Finally, N* comes more quickly in the case where congestion cost is considered than in the case where congestion is not considered.



2.3 DIFFERENT SIZE OF COMPUTERS

So far we have assumed that the computer (nodes of the network) size are all the same. Suppose each node has a different size computer. Then the running cost of the software will be different for each computer.

Let the user have a larger computer than the other nodes which supply the software through the network. Then $C\mu_1 < C\mu_2$. P_T will be greater than 1 if

$$C_i + C\mu_1 + C_{C1} > C\mu_2 + C_{C2} + C_N, \text{ or } (C_i - C_N) + (C\mu_1 - C\mu_2) + (C_{C1} - C_{C2}) > 0$$

Since $(C\mu_1 - C\mu_2)$ is less than zero, $(C_i - C_N) + (C_{C1} - C_{C2})$ should be greater than $(C\mu_1 - C\mu_2)$ in order to satisfy the condition, i.e., $U > 0$.

In checking the sign of the two terms, we find that $C_i - C_N$ is positive if installing cost (C_i) is greater than the network cost (C_N). $C_{C1} - C_{C2}$

tends to be negative, since, for the same N, C_{C1} is less than C_{C2}. As N increases, C_i and C_N decrease. C_{C1} and C_{C2} will also increase but the increasing rate of C_{C1} will be lower than C_{C2}.

Summarizing, we have the following:

1. $C\mu_1 < C\mu_2$

2. As N increases, $C_{C1} < C_{C2}$, and C_i and C_N decrease when C_N is a fixed cost.

Therefore, unless the installing cost is substantially higher than the network cost, the user will not use the network. And the critical point, where utility becomes zero, will come soon with increasing N.

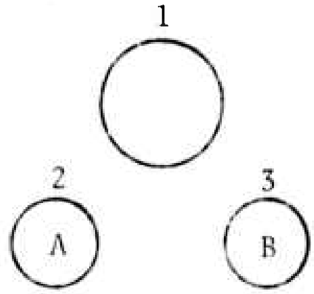
Now, suppose a user has a smaller computer than the supplier. Then, the running cost of his own computer is higher than the network's running cost.

$$C\mu_1 > C\mu_2$$

And as N increases, $C_{Ci} > C_{C2}$ and C_i and C_N will decrease. Therefore, small differences between C_i and C_N will enable the user to use the network. That is, there exists a strong tendency to use the network if the supplier's computer is large compared to the user's computer.

2.4 TWO OR MORE COMMODITIES MODEL

Another assumption to be relaxed is the number of commodities in the network. Suppose the following situation:

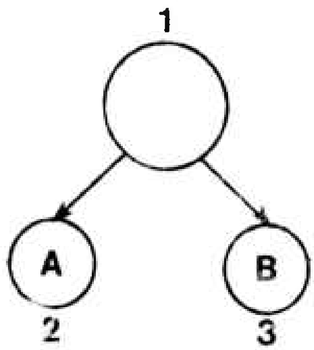


Note: 1, 2, and 3 represent the nodes of the network. A and B represent the software installed.

Here, A and B represent names of software. 1, 2 and 3 describe nodes.

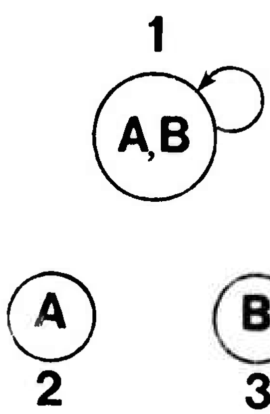
Consider user 1. As we already know, user 1 will use the network as long as his utility of the network is greater than 0. That is, his utility of the network is greater than zero if $P_T > 1$.

$$P_T > 1 \text{ if } C_i + C\mu_1 + C_{C1} > C\mu_2 + C_{C2} + C_N.$$



But since user 1 has a larger computer than the other ndoes, there is a strong tendency to install software A and B.

Since $\left.\begin{matrix} C\mu_1 < C\mu_2 \\ C_{C1} < C_{C2} \end{matrix}\right\}$, small change of C_N (or C_i) will cause user 1 to install software A and B.



Consider user 2 and 3. They have two choices of using either software A or B. For example, user 2 may use either his own computer or the network (user 1's computer) for software A. He can also use user 1's computer or user 3's computer for software B.

Let us denote U_i as the utility which user 1 will get when he uses node i's computer through the network. Then, $U_i = f(P_{Ti})$ where

$$P_{Ti} = \frac{C\mu_j + C_{Cj} + C_N}{C\mu_i + C_{Ci} + C_N}$$

As long as $P_{Ti} > 1$ the user will use node i. Therefore

$$C\mu_j + C_{Cj} + C_N > C\mu_i + C_{Ci} + C_N$$

$$C\mu_j + C_{Cj} > C\mu_i + C_{Ci}$$

is the user's boundary of using node i. Here node 1 has a larger computer than node 3. Therefore,

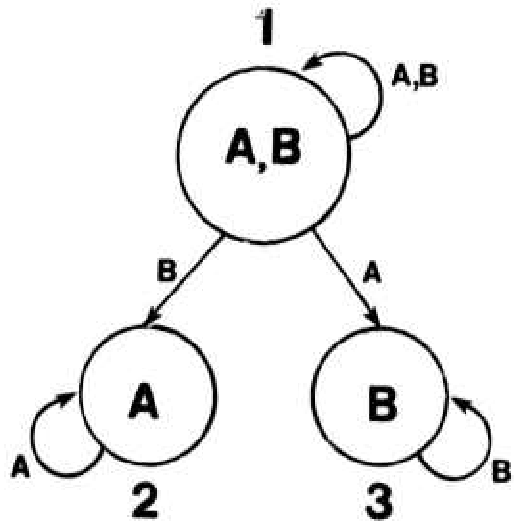
$$C\mu_3 + C_{C3} > C\mu_1 + C\mu_1.$$

Consequently, user 2 will use node 1 for software B. By the same inference, user 3 will use user 1's computer for software A.

The condition $C_i + C_{C2} + C\mu_2 > C\mu_1 + C_{C1} + C_N$ is the boundary condition for network usage to another alternative, which is to install software B in user 2. Suppose initially the above condition was met. As the number of usages of software B increases,

1. C_i and C_N will decrease (condition C_N as fixed cost). Decreasing rate may be assumed the same.
2. C_{C2} and C_{C1} will increase, but the increasing ratio is different. Since

user 1's computer is larger than user 2's computer, the increasing rate of C_{C2} will be higher than that of C_{C1}. Therefore, even if N increases, the possibility of user 2 installing software B in his computer is getting lower and lower.



3. $C\mu_1$ and $C\mu_2$ is constant, where $C\mu_1 < C\mu_2$.

Let us consider software A again from user 2's point of view. He has two choices. One is to use his own computer and the other is to use the network (node 1).

First of all, let's check user 2's utility level using the same notation used before. User 2 will use network (user 1's computer) if

$$\frac{C_{C2} + C\mu_2}{C\mu_1 + C_{C1} + C_N} = P_T > 1$$

That is, $C_{C2} + C\mu_2 > C\mu_1 + C_{C1} + C_N$. Again, since the computer of node 1 is larger than that of node 2 the following stands:

$$C\mu_2 > C\mu_1$$

$$C_{C2} > C_{C1}$$

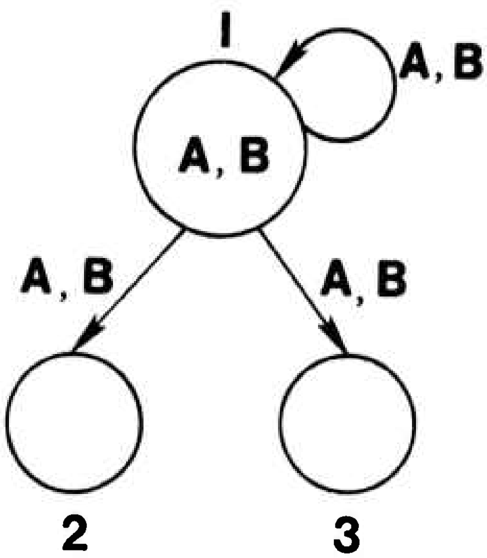
Therefore, if $(C_{C2}-C_{C1}) + (C\mu_2-C\mu_1) > C_N$ then user 2 will use the network.

Both sides are affected by changes of N, which is the number of usages of software A. As N increases:

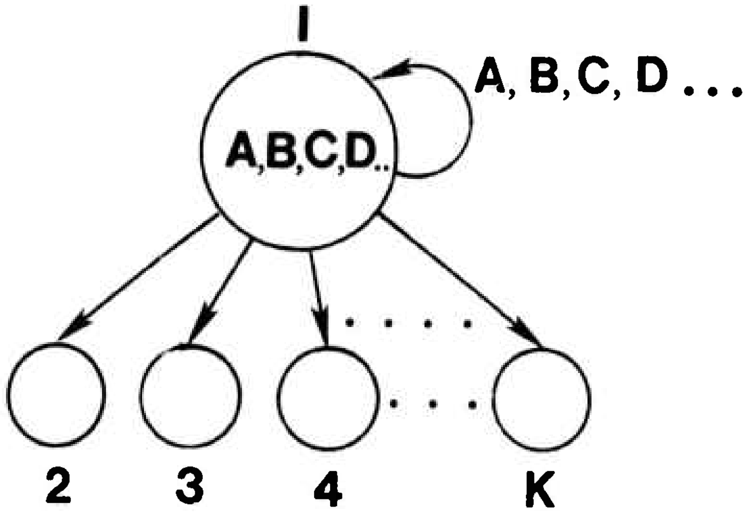
1. $C_{C2} - C_{C1}$ will increase,
2. $C\mu_2 - C\mu_1$ will also increase, and
3. C_N will be either constant or fixed.

Therefore, even if we initially have the condition where $(C_{C2}-C_{C1}) + (C\mu_2+C\mu_1)$ is less than C_N user 2 will eventually use the network for software A. From the same inference, user 3 will also use the network for software B as N becomes large enough.

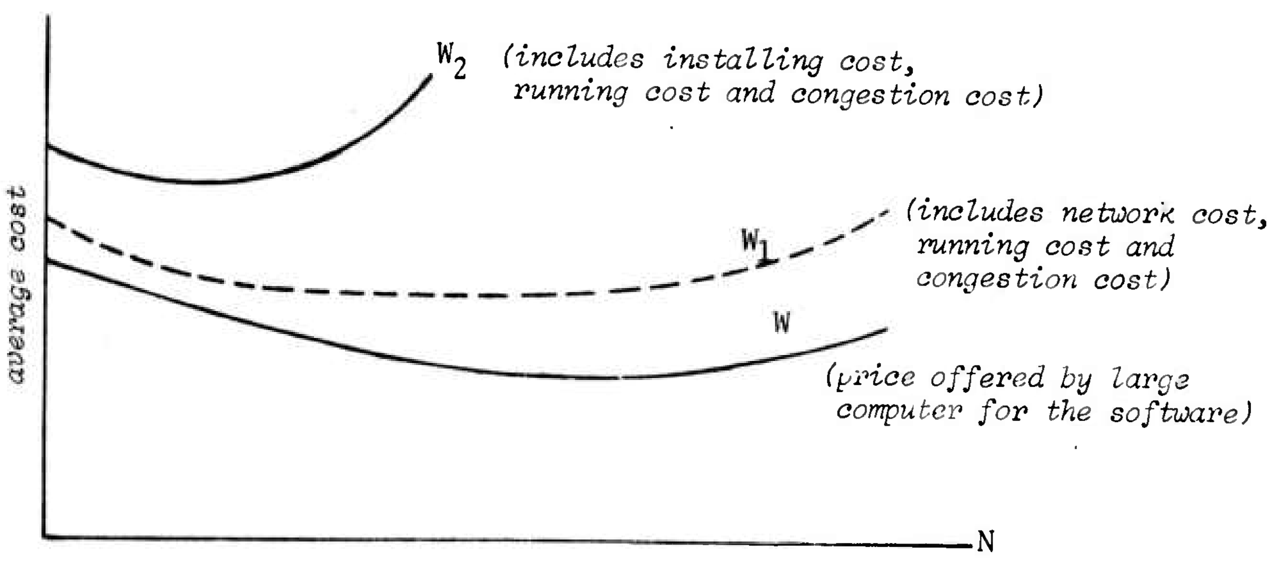
The network will have the following shape:



Our two-commodities model can be expanded into a m-commodities model. That is, eventually the network will have a STAR type network.



Our arguments that a network will eventually end up as a STAR network if it has one large node in it can be verified by the Cost Theory.



W_1 is the average cost in the case where user 2 is using the network. The difference between W and W_1 is the network cost and congestion cost.

In summary, under Assumption 5, that is, there is no congestion from the network itself:

1. The user will use the network as far as utility of network is greater than zero. That is, $U > 0$, where $U = e^k$, where

$$k = e^{\theta_T P_T / \ell n P_T} \quad \text{or} \quad k = e^{P_T 2 / \ell n P_T}$$

2. Utility can be expressed as a function of P_T which is a ratio between the cost of using his own computer and using one in the network. The user will use the network only when P_T is greater than 1.

3. P_T is a function of installing cost, running cost, congestion cost, and network cost.

4. The higher the installing cost, the stronger the tendency to use the network.

5. The larger his own computer, the weaker the tendency to use the network.

6. The larger the network cost, the weaker the tendency to use the network.

7. Determination of network type depends upon the usage of the software and the computer size the network users have. If there is one big computer, while the others are relatively small, the network will have a STAR type network. But if each node of the network has computers of the same size, then there is a certain point beyond which the user will not use the network.

2.5 CONGESTION FROM NETWORK

Now let us assume the opposite of assumption 5, that is, there is congestion from the network itself.

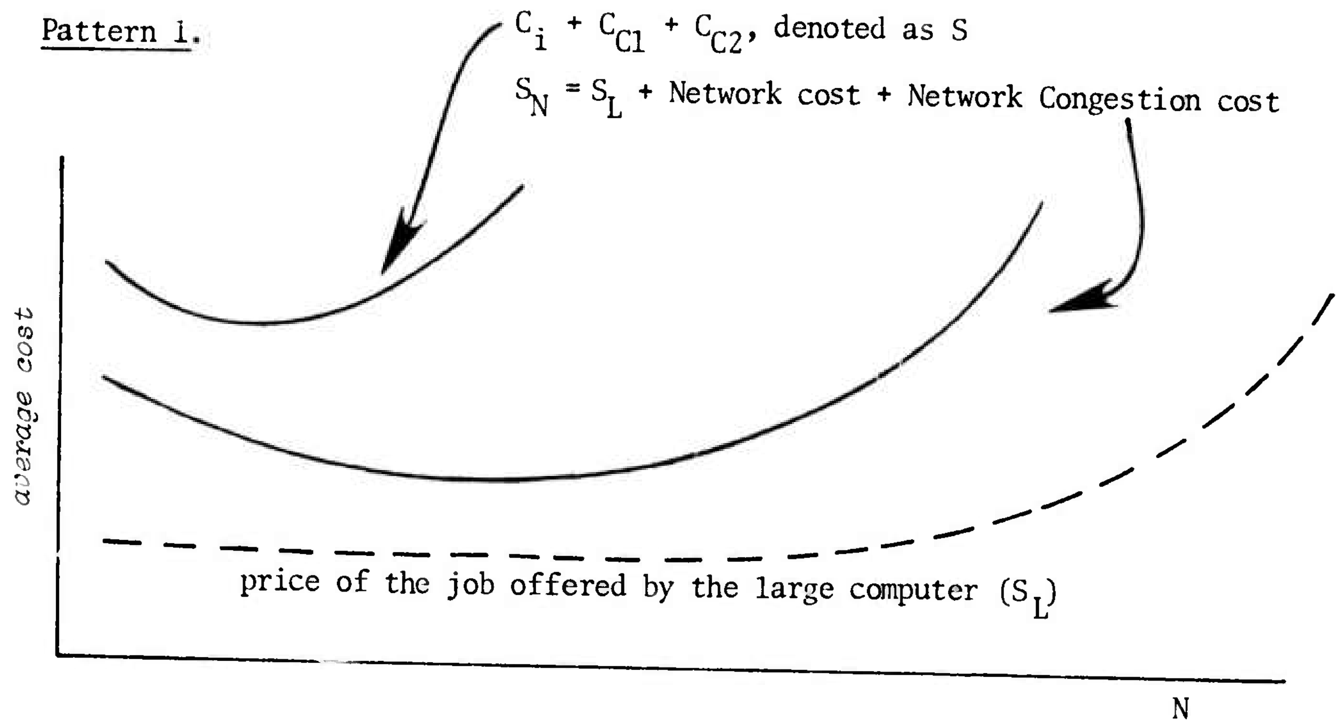
Congestion cost from the network is a function of the number of nodes, the number of software usage, user behavior using software over time, and the performance of network technology.

For simplicity, we assume that there is only one network technology, and make the following assertions:

1. Congestion cost will increase as the number of usages increases.
2. Congestion cost will increase as the number of nodes increases.

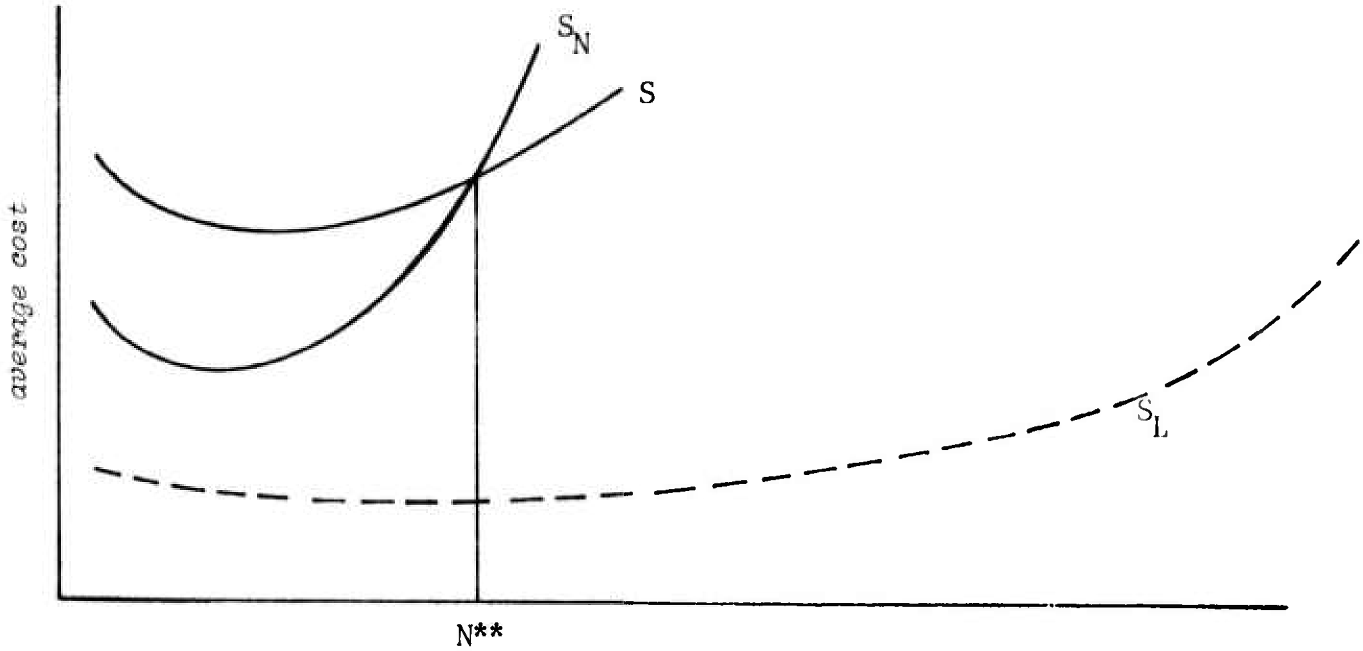
Using the same diagram used in Section 2.4, we can derive the following two patterns when considering network congestion.

Pattern 1.



In this case, throughout all values of N the utility of the network is greater than zero since $P_T = \frac{S}{S_N} > 1$. Therefore, the user will use the network.

Pattern 2.



Beyond N**, the user will not use the network since $P_T = \frac{S}{SN} < 1$ (which means that the utility of the network is less than zero).

2.6 NETWORK METHODS

The patterns considered for network congestion are determined by network methods. What we usually consider as network methods are conventional wire system (exclusive line), packet wire system, packet radio system (or The ALOHA System), and satellite system.

The cost characteristics can be summarized in the following way:

	fixed cost	operating cost
Conventional wire system	low	high
Packet wire system	high	middle
Packet radio system	middle	low
Satellite system	high	high

The following table show us another characteristic of network methods from the device point of view.

Device Characteristics of Network Methods

alternatives	interface: memory	interface: processor	modem	E/D	T/R	antenna	wire	radio channel	satellite channel
Conventional wire system			X	X			X		
Packet wire system	X	X	X	X			X		
Packet radio[1] system	X	X	X	X	X	X		X	
Satellite[2] system	X	X	X	X	X	X			X

[1] All devices checked for alternative 3 are integrated as one unit, i.e., Terminal Control Unit (T.C.U.) or P.C.U.

[2] All devices checked for satellite-based system are integrated as one unit which is called an earth station.

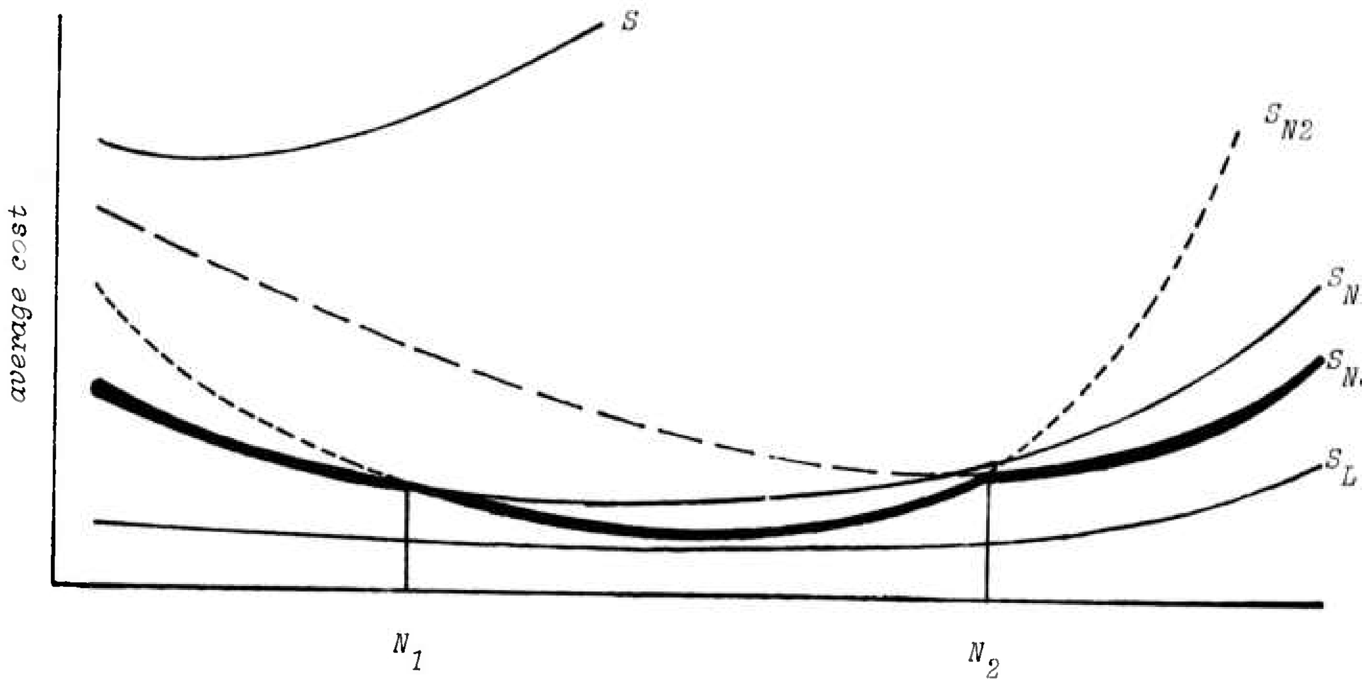
From these characterisitcs, we can draw relationships between the average costs of running software through the various network methods.

2.7 MAXIMUM UTILITY PATH[1]

Remembering that the average cost of each network method depends on the distance between the user and the software supplier, and the number of users in the network, we now discuss the maximum utility path.

In short distances, conventional wire system seems the least expensive.

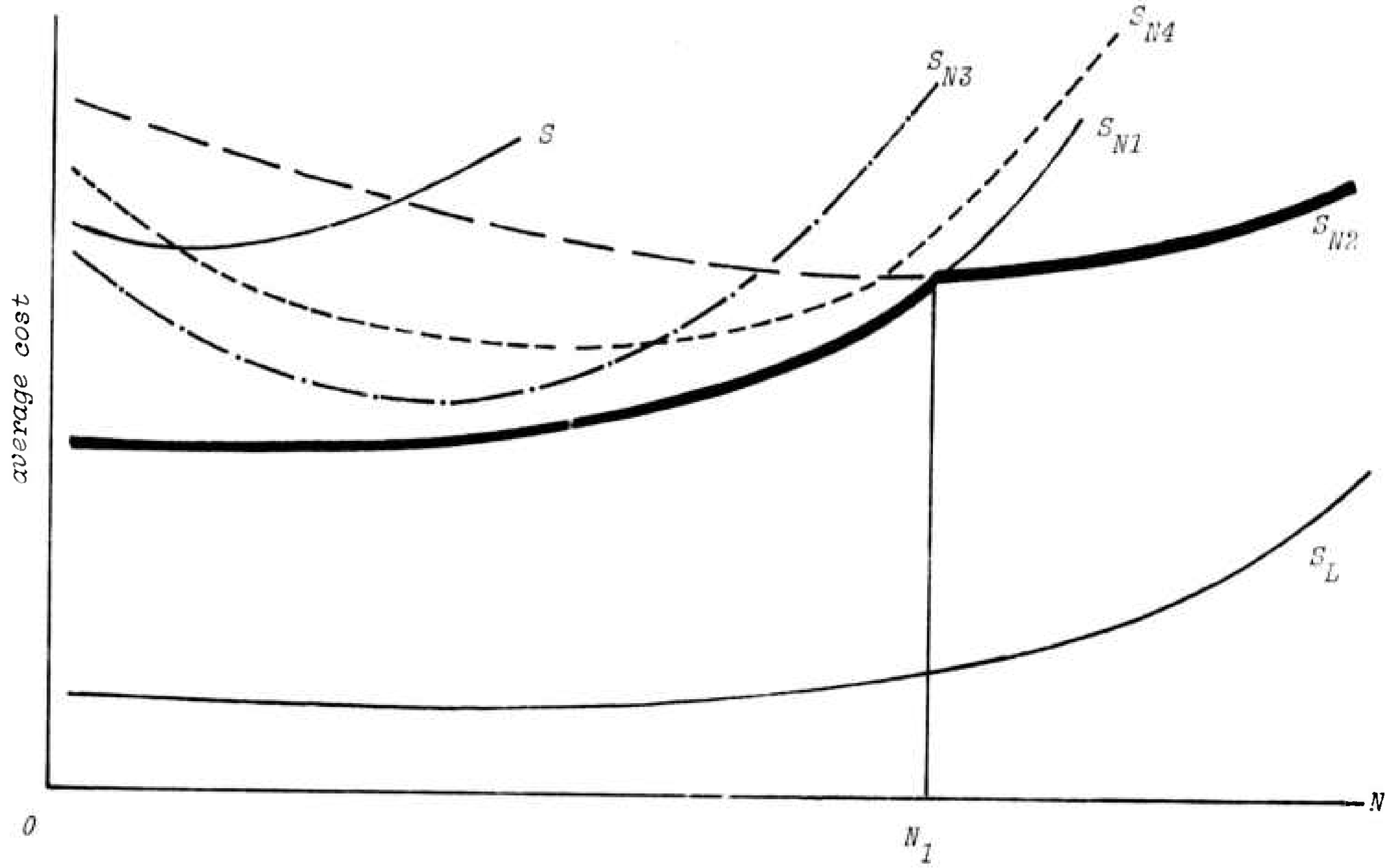
[1] All the arguments in this section will be verified from the analysis of network cost functions that are currently being studied by the author.



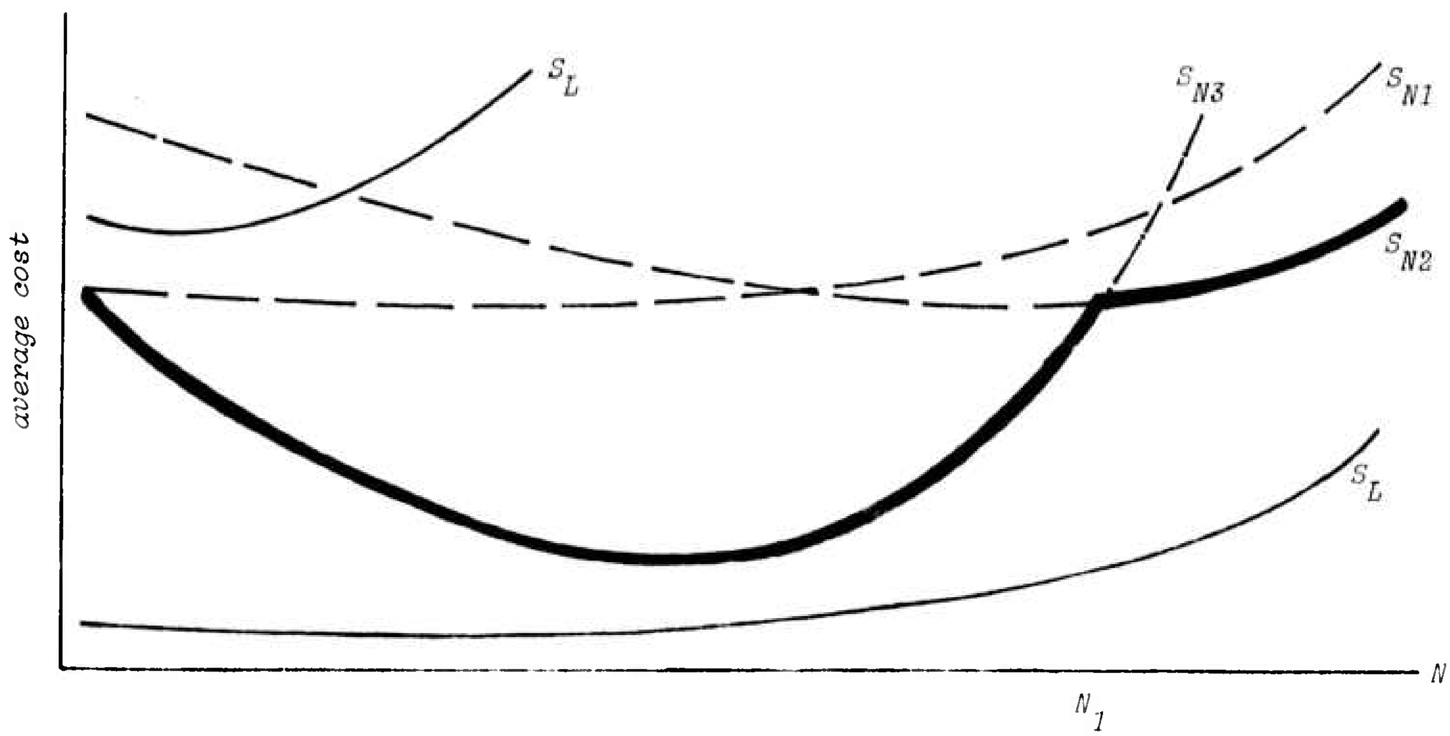
S_{N1}, S_{N2}, and S_{N3} represent the average cost of running the software through conventional wire system, packet radio system, and packet wire system, respectively. The thick line shows the maximum utility path for the network. That is, in a short distance, a user will achieve maximum utility if he follows the path, i.e., conventional wire system up to N_1; packet radio system up to N_2; packet wire system from N_2.

<u>In a long distance</u>, the shape is quite different. S_{N1}, S_{N2}, S_{N3} and S_{N4} represent the average cost of running the software through conventional wire system, satellite system, packet radio system, and packet wire system, respectively.

Again, the thick line represents the maximum utility path. For utility maximization, assuming the cost structure given below, the user should follow path, i.e., conventional wire system up to N_1 then to a satellite system.



However, since the cost of mini-computers, which functions as a node interface decreases rapidly, the following path may be feasible in the future.



S_{N1}, S_{N2}, and S_{N3} represent the average cost of running the software through conventional wire system, satellite system, and packet wire system, repsectively. The maximum utility path, in this case, is to follow a path, i.e., packet wire system up to N_1 then to a satellite system.

REFERENCES

1. C. E. Agnew, "The Economics of Time Shared Computing: Congestion Costs and Economics of Scale," Center for Interdisciplinary Research, Stanford University, October 1974.

2. "Economics of Computer Communication Networks," (Fourth Semi-annual Report), Center for Interdisciplinary Research, Stanford University, December 1974.

3. William F. Sharpe, *The Economics of Computers*, Columbia University Press, 1969.

4. *Computer-Communication Networks*, edited by Norman Abramson and Franklin F. Kuo, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973.

5. Ivan T. Frisch, "Technical Problems in Nationwide Networking and Interconnection," *IEEE Transactions on Communications*, Vol. Com-23, No. 1, January 1975, pp. 78-88.

6. Franklin F. Kuo, "Selected Papers on Computer-Communication Nets," *The ALOHA System Technical Report B74-4*, 1974.

7. Norman Abramson, "Packet Switching with Satellites," *The ALOHA System Technical Report B73-2*, 1973.